

THE ZIEGLER SPECTRUM OF A -INFINITY PLANE SINGULARITY

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Let F be an algebraically closed field of characteristic zero and let $R = F[[x, y]]/(y^2)$ be the so-called A_∞ plane singularity. We consider the theory T of x -torsion free R -modules.

The classification of indecomposable finitely generated modules in T (i.e. finitely generated indecomposable maximal Cohen–Macaulay R -modules) is well known [1]. Namely each such a module is either isomorphic to the right ideal $I_n = (x, y^n)$ of R or to the right ideal $I_\infty = xR$.

We will calculate the Ziegler spectrum, Zg , of torsion-free R -modules, i.e. a topological space (see [2]) whose points are indecomposable pure injective models of T . Because the modules I_n and I_∞ are linearly compact they are (the only) finitely generated points of Zg .

Theorem. *Each infinitely generated point of Zg is the following:*

- 1) *the ring of quotients Q of R ;*
- 1) *the integral closure \tilde{R} of R in Q ;*
- 3) *$G = F((y))$ the ring of Laurent power series.*

Furthermore one can execute the Cantor–Bendixson analysis on the Ziegler spectrum.

Proposition.

- 1) *The modules I_n , $n < \infty$ are the only isolated points in Zg ;*
- 2) *the modules I_∞ and \tilde{R} have Cantor–Bendixson rank 1;*
- 3) *Q and G have CB-rank 2.*

Thus the Cantor–Bendixson rank of Zg equals 2.

This note is an announcement of some results from a project, joint with Ivo Herzog, where infinitely generated points of the Ziegler spectrum are used to get a better (than the Auslander–Reiten quiver) understanding of the category of finitely generated R -modules.

References

1. Yoshino Y. *Cohen–Macaulay Modules over Cohen–Macaulay Rings*. Cambridge University Press, 1990.
2. Ziegler M. *Model theory of modules* // Ann. Pure. Appl. Logic. 1984. V. 26. P. 149–213.